Grade 7 Integer Addition

7.N.6	
Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially, and symbolically.	1. Explain, using concrete materials such as integer tiles and diagrams, that the sum of opposite integers is zero. [DEVELOPED] 2. Illustrate, using a number line, the results of adding or subtracting negative and positive integers (e.g., a move in one direction followed by an equivalent move in the opposite direction results in no net change in position). [ONLY FOR ADDITION] 3. Add two integers using concrete materials or pictorial representations and record the process symbolically. [DEVELOPED] 4. Subtract two integers using concrete materials or pictorial representations and record the process symbolically. [NOT DEVELOPED] 5. Solve a problem involving the addition and subtraction of integers. [ONLY FOR ADDITION]

Clarification of the outcome:

- ✦ This outcome is unpacked into two parts: addition and subtraction. Understanding subtraction depends on understanding addition. For this reason, addition should be taught first, in a lesson separate from subtraction.
- ✦ This is students' first exposure to arithmetic involving integers. Learning what integers are . . . is a grade 6 matter.

Required close-to-at-hand prior knowledge:

- ✤ Understand what integers are, why they are useful, and how to compare them (bigger . . .)
- ✤ Understand addition as a combining action.

SET SCENE stage

Provide students with the following story about a roller coaster ride.

> Mark went on a roller coaster ride. He noticed that the loading area had a sign that read 'zero elevation'. His car climbed a long way up. At the top, there was a sign that read '*you have gone up 20 metres*'. The car rolled down for a long while. At the bottom of the dip was a sign that read '*you dropped 25 metres*'. Up and down and up and down went Mark's roller coaster car. The signs along the way read: up 10 metres, down 15 metres, up 20 metres, down 10 metres, and up 15 metres. When the ride was over, the car stopped at a different place than the loading area. Mark noticed that the elevation sign at the stopping area was damaged, making it unreadable. Mark wondered if he was higher or lower than where he had started the ride.

The problem task to present to students:

Organize students into groups. Ask them to answer Mark's question using whatever method makes sense to them.

Ask for and discuss students' approaches to a solution but do not indicate what the correct answer is yet. Ask students if the situation involves addition of integers. Ensure students realize that it does. Represent the situation by a number sentence involving bracket notation (see note below) by writing it on the board:

(+ 20) + (-25) + (+10) + (-15) + (+20) + (-10) + (+15) = ?

Tell students that they will be learning about integer addition and that they will revisit Mark's question later in the lesson.

Comments:

The purpose of the task is to expose students to a situation that concerns integer addition and to introduce them to how integer addition might be done.

Note:

One of the issues with understanding integer arithmetic is the notation that is used. The adult world uses flat notation [e.g.: -2 + 7 = 5.] The teaching issue with this notation is that '+' and '-' can refer to an arithmetic operation (add or subtract) or to a number (a positive or negative integer). This can easily result in student confusion. Two notations specifically intended for teaching purposes have been used:

- Raised notation [e.g: $^{+}2 + {}^{-}3 = {}^{-}1$].
- Bracket notation [e.g.: (+2) + (-3) = -1]

It does not matter if raised or bracket notation is used for teaching purposes. The switch to flat notation will have to occur at some point. Then, typically, students' comfort level drops. This lesson uses bracket notation to begin the development of integer addition, partly because there is a technical advantage to using it. Computer keyboards make bracket notation simpler to write and the result easier to read. Then the lesson switches to using flat notation.

DEVELOP stage

Activity 1: Addresses achievement indicator 2.

Have students play the game, 'ZERO ME'.

The game: 'ZERO ME' (Number of players: 2 to 4)

Requirements:

- A number line that goes from -20 to +20 (only integer values marked on it)
- A deck of normal playing cards
- Some type of different playing pieces to mark player positions on the number line

Rules:

- Each player places his/her playing piece on 'ZERO' on the number line.
- Players take turns turning over a playing card from the deck of playing cards. The player moves his/her playing piece the number of times indicated by the card (for 2 to 9: as indicated by the number on the card; Ace: 1 or 11; face cards: 0 or 10). If the card is red (a heart or diamond), the playing piece moves in the negative direction on the number line. If the card is black, the playing piece moves in the positive direction. EXAMPLE: Suppose a playing piece is at -3 and the player turns over the 7 of diamonds. The player moves his/her playing piece 7 places left, stopping at -10 on the number line.
- If the playing piece cannot move any further left or right (past -20 or +20), it stays at -20 or +20 until it is able to move either right or left again.
- The winner is the player whose playing piece returns to zero first.

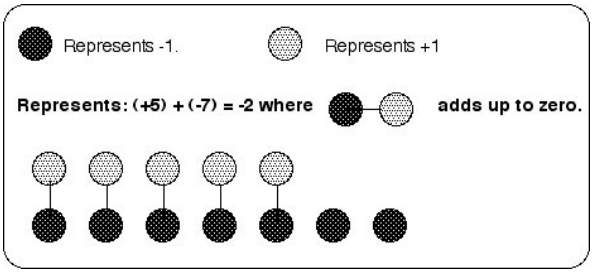
Ask students what arithmetic operation (+, -, x, or ÷) is involved in ZERO ME. Ensure students realize that the game involves addition of integers.

Activity 2: Addresses achievement indicator 3 (if students want) and 5.

- ✦ Present a situation about gaining and losing money (e.g.: Harry has 15 dollars in his pocket. He spends $5). Ask students if the situation can be represented by integer addition. Ensure they realize that it can by thinking of it as, for example, starting with positive 15 and then combining that with a loss of 5 (adding -5). Ask them to represent the situation with integer addition. Ensure they can as, for example: (+15) + (-5).
- ✦ Present a variety of gaining and losing money situations. Ask students to represent each situation with integer addition and then to figure out the answer using whatever method makes sense to them.

Activity 3: Addresses achievement indicators 3 and 5.

- Discuss electric charges (e.g. batteries have a positive and a negative terminal) and protons and electrons. Have students research protons and electrons. Here is one useful website for that: **Atom Models**
- Tell students that the two types of charges (protons and electrons) can be represented with counters having red (for example) on one side and black (for example) on the reverse side. Discuss that agreement is needed on which colour represents positive and which negative.
- Organize students into groups. Provide each group with sufficient two-colour counters. Ask students to solve about four integer addition problems involving the electric charge context (e.g.: A charge of +5 is combined with a charge of -3. What is the resulting charge?). Ask students to model each problem and determine the solution using the counters. Note that you may have to assist them to realize that matching a negative with a positive counter cancels the two counters. After each problem is solved, use bracket notation to represent the integer addition [e.g.: (+5) + (-7) = -2].

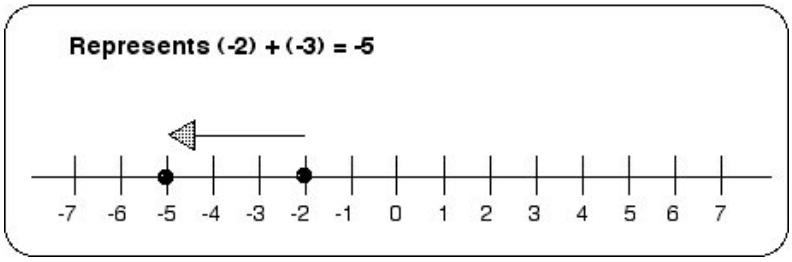


Activity 4: Addresses achievement indicators 1 and 3.

Present students with special pairs of integers (opposites) such as +2 and -2. Ask them how the two integers in each pair are related. Tell them such pairs of integers are called opposites. Ask students what the answer is if two opposites are added. Ask students to explain the result by using a model (e.g. number line, having and owing money, two-colour counters, above and below par in golf, etc.) and to represent the situation with an addition number sentence (e.g. (-5) + (+5) = 0). Ensure they realize the sum of opposites is always zero.

Activity 5: Addresses achievement indicators 2 and 3.

Revisit the game ZERO ME. Present a variety of situations (neg + neg; pos + neg, etc.). Have students use a number line to determine the result and represent each situation with bracket notation [e.g. (-2) + (-3) = -5].



Activity 6: Addresses achievement indicator 5. (developing short cut method)

- List some of the addition results from the previous activities [e.g. (-2) + (-3) = -5] in a systematic way. A table such as the one shown below is very useful for the purpose.

pos + pos	neg + neg	pos + neg	neg + pos
(+5) + (+3) = +8	(-5) + (-3) = -8	(+7) + (-2) = +5	(-7) + (+3) = -4
(+1) + (+2) = +3	(-2) + (-3) = -5	(+3) + (-3) = 0	(-2) + (+2) = 0
(+4) + (+5) = +9	(-1) + (-4) = -5	(+2) + (-5) = -3	(-1) + (+3) = +2

- Ask students if they believe the listed results. Ensure they do because of the contexts/ materials used to obtain the answers. Ask students to look carefully at the questions and the answers and then to figure out a short cut method for adding integers. Ensure they realize some version of the following method:
  - ☑ If the integers have the same sign, add the whole number parts, and keep the sign.
  - ☑ If the integers have different signs, subtract the whole number parts, and keep the sign of the larger whole number.

Note:

While the above short cut method may look familiar to the reader from their school days, the distinction is that the students figure out the short cut method AFTER they have obtained answers to addition questions by using contexts/materials. Thus they may actually understand why integers are added in the way that they are.

Activity 7: Addresses achievement indicators 2, 3, and 5 & practice.

Provide students with a mixture of about eight integer additions (written using bracket notation) that involve all four cases (p + p, n + n, p + n, n + p). Ask students to use the short cut method to figure out the answers and then to use a number line or two-colored counters to confirm the answers for some of the questions.

Activity 8: Addresses achievement indicators 5.

Revisit SET SCENE by writing the number sentence on the board.

(+ 20) + (-25) + (+10) + (-15) + (+20) + (-10) + (+15) = ?

Discuss how such complex integer addition might be done. Ensure students realize complex addition is simply a whole bunch of addition of two numbers done over and over again. Ask students to obtain an answer to Mark's question by using the short cut method. Ensure they obtain + 15 as the answer.

Activity 9: Addresses achievement indicator 5 & practice. (the shift to flat notation)

- Present an integer addition that involves a positive add a positive. Represent the question with bracket notation [e.g. (+3) + (+5)]. Discuss if there might be another way to write the addition question. Help students by discussing non-mathematical situations where there are different ways of saying the same thing (e.g. hello/hi; big/large, etc.). Ensure students realize that, 3 + 5, for example, is another way of writing (+3) + (+5). Ensure they realize that a positive is implied if no sign is there and that the '+' indicates addition. Mention that they are learning another way because that is how integer arithmetic is written by engineers, scientist, technicians, etc.
- Present an integer addition that involves a negative add a positive. Represent the question with bracket notation [e.g. (-5) + (+2)]. Ask students to write it in another way. Discuss. Ensure students realize that, -5 + 2, for example, is another way of writing (-5) + (+2). Ensure they realize that a positive is implied if no sign is there, that the '+' indicates addition, and that the '-'indicates a negative number.
- Present an integer addition that involves a positive add a negative. Represent the question with bracket notation [e.g. (+5) + (-2)]. Ask students to write it in another way. Discuss. Ensure students realize that, 5 - 2, for example, is a way of writing (5) + (-2). Discuss that '-' can indicate addition of a negative. **Help students with the implied addition by reading 5 - 2 as "positive 5 add negative 2". Mention that we can also read 5 - 2 in a subtraction way as positive 5 subtract positive 2, but we are not ready for that yet. We have to wait until we learn about integer subtraction.**

  NOTE:

  This situation has potential for confusion. Mention that we looked at 5 - 2 in a subtraction way since grade 1 but we can also look at it as addition of integers. Either way, the answer is 3.
- Present an integer addition that involves a negative add a negative. Represent the arithmetic with bracket notation [e.g. (-5) + (-2)]. Ask students to write it in another way. Remind them about the other way for (+5) + (-2). Ensure students realize that, -5 - 2, for example, is a way of writing (-5) + (-2). Discuss again that '-' can indicate addition of a negative. **Help students with the implied addition by reading -5 - 2 as negative 5 add negative 2". Mention also that we can read -5 - 2 in a subtraction way as negative 5 subtract positive 2, but we have to wait until we learn about integer subtraction. Either way, the answer is -7.**
- Ask students if the short cut method for adding integers they figured out earlier might work for the way of writing integer addition that engineers and such people use. Discuss. Do an example of each case with them (e.g. 2 + 3; -2 + 3, 2 - 3; -2 + 3) so they realize the short cut method still works.
- Provide students with a mixture of about eight integer additions that involve all four cases (p + p, n + n, p + n, n + p) and that uses flat notation (e.g. -7 - 8; -6 + 4). Ask students to use the short cut method to figure out the answers.

Activity 10: Assessment of teaching.

Provide students with two integer additions written using flat notation, where one question involves a negative add a positive [e.g. -3 + 7] and the other a negative add a negative [e.g. -3 - 7]. Ask students to:

- determine the answers, using the short cut method
- explain why each answer is correct, using a number line or two-colored counters.

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

An example of a partially well-designed worksheet follows.

The worksheet contains a sampling of question types. More questions of each type are needed.

The MAINTAIN stage follows the sample worksheets.

Question 1.

Write each statement as an integer addition question using both bracket notation and adult (flat) notation and then determine the answer.

a) A gain of 5 and a loss of 3.

b) A gain of 7 and a gain of 3

c) A loss of 4 and a loss of 2

d) A loss of 5 and a gain of 2

Question 2.

Write each statement as an integer addition question using both bracket notation and adult (flat) notation and then determine the answer.

a) A positive charge of 5 combined with a negative charge of 6.

b) A positive charge of 2 combined with a positive charge of 3.

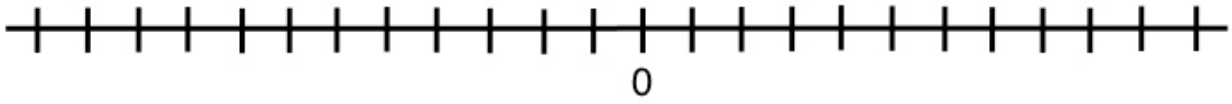
c) A positive charge of 4 combined with a negative charge of 1.

d) A negative charge of 3 combined with a negative charge of 2.

Question 3.

Use a number line to obtain the answer to each integer addition.

a) -2 + 4



b) 4 - 5

c) -1 - 3

d) 2 + 1

Question 4.

Determine the answer to each integer addition.

a) -10 - 3

b) -10 + 3

c) 10 - 3

d) 10 + 3

MAINTAIN stage

Mini-task example

Every so often:

- Present four integer addition questions (one for each case: pos + pos; pos + neg; etc.) Ask students to determine the answers.

Rich-task example

Have students solve the following problem by representing what is going on with **integer arithmetic number sentences** and then doing the indicated arithmetic.

> *Joe plays golf a lot. He played in a tournament that involved five rounds of golf. His five scores were: 2 above par, 4 below par, 1 above par, 3 below par, and 5 above par. The rules of the tournament require each player to add up all five scores. This is total A. Then each player has to add his best and worst scores and divide the sum by 2. This is total B. Each player figures out his/her final score for the tournament by calculating: total A - total B. What is Joe's final score for the tournament?*

Comments

This is a rich-task because it involves solving a complex problem.